Time value of money, NPVs, IRRs
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Understanding of time value of money and present valuation is the very backbone of financial mathematics. For any student in finance, it is crucial that there is a very clear conceptual understanding and computational ability in computing present and future values.

> **Jargon:** Discounted cashflow analysis, present value analysis, time value of money – all these phrases refer to the same analytical techniques concerning cashflows spread over time.

Understanding time value of money:

To understand time value of money, it is most important to understand the role of interest in financial transactions. There is interest in most financial transactions: for example, giving or taking of a loan, investing in or issuing a bond, buying a government security, investing money in any mode of saving, etc. All these transactions are those that carry a fixed rate of return, commonly termed as "interest". There are lots of other transactions that may not carry a fixed rate of return – such as investing money in equity stocks, or gold, or any other asset for that matter. Each such investment is intended to produce some rate of return, though what such rate of return would be is not contractually fixed.

Thus:

- All financial transactions that involve a period of time are designed to produce a rate of return to the investor, and a cost to the investee.
- Whether the rate of return is contractually fixed or not depends on the type of investment. Usually, ownership or equity type investments do not carry fixed rate of return. Usually, debt type contracts carry fixed rate of return.
- For the sake of convenience here, we will term fixed rate of return as "interest".

Irrespective of whether I earn a fixed rate or a variable rate, if I am investing money for a period of time, I would expect that my money grows over period – that is, I am expecting a rate of return.

Why does money have to have a rate of return? One common reason is inflation. As money is not of value by itself – it is only a medium of exchange – inflation keeps depleting the "intrinsic" value of money over time. Therefore, if I am not earning a rate of return on my money over period, my money is actually depleting in value.

Simple and compound interest:

The idea of simple interest is – interest continues to accrue over a period of time at a given rate.

For instance, if I invest $ 1000 at interest @ 10% per annum, I earn $ 100 over 1 year, and $ 200 over 2 years, and $ 300 over 3 years and so on. If I invest $ 1000 today, and get it back after 5 years, I will get a total of $ 1500, including interest of $ 500.

The generalized formula for simple interest is:

SI = P(r)(n) … (1)

Where, P = Principal invested today
SI = Simple Interest
r = Rate of interest
n = time period

However, in most financial transactions, particularly those involving longer periods of time, it would be impractical to be limited to the over-simplified concept of simple interest. Here is the reason – in the example above, if I actually invested money for 1 year instead of 5 years, I would get an interest of $ 100. I can now invest $ 1100, and get an interest of $ 110. Now, I have $ 1210 in pocket, which I can now invest and earn an interest of $ 121, and so on.

The moment we either consider interest received and reinvested, or automatically reinvested in the transaction by computing interest on interest as well, we are *compounding* the interest. In case interest is compounded, it is as if the recipient received and reinvested the interest. In a way, compounding of interest is a compensation for the investor not actually receiving interest periodically.

Unless a transaction actually pays out interest (so that the recipient may either reinvest or otherwise enjoy it), compounding of interest is almost unquestionable practice in the world of finance.

The generalized formula for compound interest is:
$A = P (1+ r)^n$ … (2)
CI = A – P

Where, A = Amount, i.e principal + interest
P = Principal invested today
CI = Compound Interest
r = Rate of interest
n = Time period

Compounding frequency:

Of crucial importance in compounding of interest is compounding frequency, that is, how often do we compound interest.

Example 1

Let us say, I have $ 1000 to invest, and the rate of interest is 10% per annum, for a tenure of 5 years.

- Depending upon the terms agreed with the borrower, I might compound interest every year – so I compound it 5 times during the 5-year term
- Or, I might compound it every quarter – so I compound it 20 times, at the rate of interest of 2.5% per quarter.
- Or, I might compound it every month, at the rate of interest of 10%/12 per month, for 60 months, that is, 60 times over the 5-year term.

Needless to say, the values in each case will differ. Hence,

- Compounded value depends on the rate of compounding
- Compounded value also depends on the frequency of compounding

Tip: How do you do these computations on normal, non-financial calculators? Most normal calculators have facility for doing a continuous operation, that is, multiplication/division/addition/subtraction. The exact sequence of keys depends on the calculator's internal logic. However, generally, one of the following two approaches works:

- In first computation above, there is a continued multiplication of 1 by 1.1 (1+10%). Setting 1.1 as the constant multiplier, one puts 1.1 X X 1, and then every time, one presses (=) key, one gets the compounded number. So, press it 5 times, and get the result.
- Alternatively, put 1X 1.1, and then keep pressing presses (=) key, and see if the calculator gives compounded values.
- These tricks are based on the logic of your calculator – so you need to try either way to make your calculator work. Most calculators do, however, have a constant operation facility.

More frequent compounding:

As we noted in Example 1, the compounded value at the end of 5 years was higher with higher compounding frequency. What if I increase the compounding frequency even more? Obvious answer is, the compounded value will keep going up.

If there a justification for more frequent compounding, say, per day? Obviously, one cannot envisage a traditional loan transaction where interest is received on something like every day. However, active players in financial world, such as banks, traders in financial securities, etc. do actually invest money on overnight basis. For example, it is quite common for banks to invest money overnight. Traders in stocks or bonds keep churning money several times during a single trading day. Money is liquidity, and active players may be taking advantage of liquidity with very short intervals.

Hence, there may be a case for more frequent compounding. Once again, as we keep reducing the compounding period and increasing the compounding frequency, the compounded values keep going up. This brings us to a compounding limit, called continuous compounding.

Continuous compounding:

If we keep increasing the compounding frequency, say, to compounding infinite number of times during a given term, we hit the limit or the maximum value that compounding can produce. This is called continuous compounding, which is like compounding every moment.

The generalized formula for continuous compounding is:

$FV = PV.e^{rn}$ … (3)
$CI = FV – PV$

Where, e = exponential
n = tenure

Example 2:
Example 1, re-done based on continuous compounding, leads to a value of 1648.721.
Note: Value of e is 2.718282.

Future value of money and present value of money:

Future value:

In our compounded value examples, we took the case of loan transaction and computed interest on it at a particular rate of interest. In many cases, we may not have any actual loan, and may still like to find out what a compounded value would be in future, if a certain rate of return (not necessarily interest) were assumed. That is what we call the *future value of money*, that is, the value that money will acquire in future, if compounded at a given rate of return.

The generalized formula for future value of money is the same as that for compounded value of money, except that here, the rate is not necessarily the contractual rate of interest:

$FV = PV (1+r)^n$ … (4)

Where, FV = Future Value of money
PV = Present Value of money
r = interest rate (assumed/ actual)
n = time period

In any computation of future value of money, it is quite obvious that there is a rate of return and a compounding frequency. The rate of return may be the actual rate that the investor is expecting, or the opportunity cost, that is, how much return would he have made had he invested in some other mode of investment.

Present value:

If future value is the value of money that one has in present, in future, present value is exactly the reciprocal – that is, value of money that is expected in future, today. Once again, as in case of future value, there is a certain rate of return inherent in the computation, and a certain compounding frequency.

Generalised formula for computing present value:

$PV = FV/ (1+r)^n$ … (5)

Discounting and Discounted value:

As we compute the value of present money in future, we are compounding it. As we compute the value of future money at present, we are *discounting* it. That is, discounting is the opposite of compounding. In like terminology, present value is also known as *discounted value*.

The concept of present value is quite simple. Let us understand this with the help of an example.

Example 3:
Let us suppose I expect a cashflow of $ 1000 at the end of 5 years, and my rate of return is 10%, with an annual compounding frequency. The present value comes to $ 620.92.

Note the following:

- If an amount of $ 620.92 is invested, at a 10% rate of return, for a period of 5 years, annually compounded, the sum would add exactly to $ 1000 at the end of 5 years.
- The present value is obviously dependent on the rate of return, that is, the discounting rate, and the frequency of discounting.
- More the discounting rate, less will be the discounted value.
- Higher the discounting frequency, less be the discounted value.
- Needless to say, the discounted value for a given rate of discounting will be the least if discounted on continuous discounting basis.

Generalized formula for discounting on continuous discounting basis:

$PV = FV/ e^{rn}$ … (6)

Where, r = discounting rate
n = years

Finding rate of return from present and future values:

In the preceding sections, we have gone through easy computations of (a) computing future value from present value, at a given rate of return; and (b) computing present value from future values, again, at a given rate of return. Needless to say, if both the present value and the future value are known, it would not be difficult to find the rate of interest as well.

Example 4A
Assume I am lending $ 1000 for 1 year. At the end of the year, I get a value of $ 1100.

We really do not have to do much formula-writing here, as anyone with slightest computational ability can say that I am earning interest at the rate of 10%. (**Teaser question** - is it 10% simple or 10% compounded?). However, we will still put it into a formula to be able to advance into the forthcoming examples. Hence:

$FV = PV (1+r)^n$ … (4)
Or
$1100 = 1000 (1+r)^1$
hence, 1+r = 1100/1000
or; r = 1.1 - 1
Interest = .1 or 10%

Example 4B
Assume I am lending $ 1000 for 2 years. At the end of 2 years, I get a value of $ 1200.

To any one not initiated in finance, here also, one may say, the rate of interest is 10%, as I am earning 20% over 2 years. However, that would be simple interest, which, as we said before, is almost never used in real life financial transactions pertaining to reasonably long enough period of time.

So we need to compute the rate of interest by putting the numbers into the equation:

$FV = PV (1+r)^n$ … (4)
Or
$1200 = 1000 (1+r)^2$
or; sq rt(1200/1000) = (1+r)
or; 1+r = +/-1.0954
or; r = +0.0954 or -2.0954

Being a quadratic equation, the 2 mathematically possible answers given by the equation are: interest of: +9.54%, or of – 209.54%. Since one answer is negative, and interest rates are normally not negative, we ignore the negative answer as unreal, and accept the positive answer.

Hence, the rate of interest here is 9.54%

Example 4C
Assume I am lending $ 1000 for 3 years. At the end of 3 years, I get a value of $ 1300.

Following the same process as we did in Example 4B,

$1300 = 1000 (1+r)^3$
or; $(1+r)^3 = (1300/ 1000)$
or; $r = 9.2\%$

Hence, the interest rate is 9.2%

Use of interpolation device for computing rate of return:

Example 4D

Assume I am lending \$ 1000 for 3 years. I receive the following amounts: at the end of year 1, \$ 400, at the end of year 2, \$ 400, and at the end of 3 years, \$ 400.

In other words, instead of a getting a single payment as at the end of n number of years, I am getting a series of payments. Putting these numbers in the equation, we have:

$\$ 1000 = 400/ (1+r)^1 + 400 / (1+r)^2 + 400/ (1+r)^3$

This type of an equation will be difficult to solve by direct computation. Hence, we use an interpolation device starting with a guess rate and then perfect the computation until we are able to get the precise rate.

The basis of the interpolation device is the equation that we have used time and again above, that is:

$FV = PV (1+r)^n$ … (4)
Or
$PV = FV/ (1+r)^n$

What this means is that if we compound a present value at the actual rate of return, we get exactly the future value, or vice versa, if we discount a future value(s) at the actual rate of return, we actual get the present value. This means, we can discount future cashflows at any assumed rate or guess rate, see the gap between the discounted value and the present outflow (one more jargon – the gap is called *net present value*; we discuss net present value later in this chapter), and then narrow down on the computation by altering the guess rate till we are able to zero down the net present value.

Does this mean, we will keep fumbling for the right rate till we experiment with innumerable guess rates? Not really. There is a quick technique of interpolation which goes as under:

- We find net present value at some guess rate. This may be any guess rate, say 10% ($Rate_L$). We call it NPV_L.
- We then find net present value at one more guess rate, say, a higher guess rate of 11% ($Rate_H$). We call it NPV_H.

- Note that our objective is to zero the NPV. Hence, if NPV_L is positive, we have to so increase the guess rate as to reach a zero NPV, or if NPV_L is negative, we have to so reduce the guess rate to reach a zero NPV.
- We assume, though it is not correct, that the relation between the discounting rates and NPVs is linear[1]. So, if NPV has been reduced (or negative NPV increased) from NPV_L to NPV_H, how much do we need to increase the discounting rate to make the NPV zero? This would be our third guess rate, that is:

  Third guess rate = $Rate_L + NPV_L / (NPV_H) * (Rate_H - Rate_L)$
- Then, we compute a fourth guess rate using the results of the third guess rate and the second guess rate as the basis, and then a fifth guess rate using the results of the third and the fourth guess rate.
- We hope to be able to reach an answer precise to a few decimal points in 5 or 6 attempts.

Tip: The working below may be done on normal calculators. Use the continued division facility. In case of discounted values at 10%, the divisor is 1.1. In case of 11%, it is 1.11, and so on.

Sum of money invested 1000

Inflows received as under

Year	Cash inflow	Discounting at Guess rate	Discounting at Guess rate	Interpolated guess rate	Interpolated guess rate	Interpolated guess rate	Interpolated guess rate
		10%	11%	**9.695%**	**9.701%**	**9.701%**	**9.701%**
1	400	363.6364	360.3604	364.6467	364.627	364.6274	364.6274
2	400	330.5785	324.649	332.4181	332.3822	332.3829	332.3829
3	400	300.5259	292.4766	303.038	302.9888	302.9897	302.9897
Total PV		994.7408	977.4859	1000.103	999.998	1000	1000
NPV		-5.2592	-22.5141	0.102852	-0.002	-1.7E-07	0

Note that the interpolated rates in bold in the Table above have been arrived at using the formula discussed earlier. Result, the rate of return is 9.701%.

Internal rate of return:

The rate that we just obtained in Example 4D may also be called the *internal rate of return* or *implicit rate of return*. The word internal or implicit is only to state that on the face of it, the rate was not explicit. It was hidden in the way the numbers were. There is nothing imaginary or notional about this rate – this is the rate of return produced by the transaction in Example 4D, exactly as the rates computed in any of the earlier examples in this Chapter.

[1] The relation between discounting rates and NPVs is not linear – it is convex. Try a sample calculation computing the NPV of any future value, at different discounting rates. Every time the discounting rate is increased by say 1%, the resulting decrease in NPVs becomes lesser and lesser. By the way, if the relation between discounting rates and NPVs was linear, there would have bee n no need to interpolate, as it would be possible to get the exact answer having done only one guess rate computation.

By definition, IRR is the rate where NPV = 0
That is to say:

$NPV = 0 = CF_0 - (CF_1/(1+r)^1 + CF_2/(1+r)^2 + \ldots\ldots + CF_n/(1+r)^n)$
Or; $CF_0 = (CF_1/(1+r)^1 + CF_2/(1+r)^2 + \ldots\ldots + CF_n/(1+r)^n)$ …(7)

Where, IRR = Internal Rate of Return
NPV = Net Present Value
CF = Cash Flows

Or; writing the above series in sigma notation,

IRR= r, such that $\sum_{i=1}^{n}[CF_i/(1+r)^i] - CF_0 = 0$ …(8)

Here are some quick notable points about IRRs:

- Is IRR the same as rate of interest? IRR is actually a rate, and as the name implies, the rate of return inherent in a transaction. If the transaction in question is a loan, such rate is rate of interest. If the transaction in question is not a loan, for instance, investment in a property, it is not appropriate to use the word "rate of interest". Hence, IRR is more generic.
- In a loan, if the rate of interest is 10%, will it be okay to say that IRR is 10%? Generally speaking, our answer would be, yes. Of course, there are certain details that we will like to introduce at we go.
- Does that mean, in loan transactions, there is no need to compute IRRs? If the rate of interest in a loan is explicit, and there are no other significant cash inflows or outflows than the payment of interest or principal, it may not be necessary to compute IRRs. However, there are details that we introduce later.
- There is a common notion that the computation of IRR is based on an assumption that every cashflow is reinvested, and reinvested at the same rate as the IRR. Do you agree? Our short answer, at this stage, would be – there is no basis for this notion. This is indeed a misnotion, and we explain later what this actually means.

Net present values

We have discussed future values and present values at the start of this Chapter. If we consider a series of cashflows, and compute present values of all the cashflows at a particular discounting rate, and then sum up the present values, the result is called *net present value*. The reason why we use the word *net* is that a series of cashflows would most likely have an outflow at the beginning, followed by several inflows, or vice versa. So, in effect, we are netting out the present values of the positive and the negative cashflows, and hence the term net present value.

Mathematically

NPV = $\sum_{i=1}^{n}[CF_i/(1+r)^i]-CF_0$ …(11)

Where r is the discounting rate for computing the NPV.
A key input in computing the net present value is the discounting rate. This point needs elaboration, but before coming to that, let us first spend some time discussing methods of computing NPV, and take some examples.

How to compute NPV:

The simplest and universally applicable method of computing NPV is to take each cashflow, and discount it at the given discounting rate for the time period after which the cashflow occurs. The definition of the "time period" for discounting each cashflow is the same as in case of IRRs. For instance, if a cashflow occurs after 1 year, I might say, it is received after 1 year, or after 12 months. In the former case, I will use an annual discounting rate, and discount the cashflow for 1 period; in the latter case, I will use the monthly discounting rate, and discount the cashflow for 12 periods. The result will not be the same – a point that we have discussed several times earlier.

Having computed the present value of each cashflow, we sum up the present values, and that is the net present value.

Example 6
Let us suppose I give a loan of $ 1000, and recover the 36 monthly instalments of $ 34 each. Now, I want to compute the NPV of the series of cashflows at a discounting rate of 10% p.a.

Tips for working on normal calculators:

Since the annual discounting rate is 10%, its monthly equivalent is 10%/.12, or 0.8333%. Hence, we will divisor 1.083333. If do this computation 36 times. The results of 36 iterations may either be added manually, or if the calculator has a facility for storing the result in memory, in the calculator memory.

To the sum of the 36 present values, multiply 34, being payment each period. This is the aggregate present value. Now get the NPV.

We get a net present value of 53.702.

What does the NPV imply?

The computation in Example 6 above resulted into NPV of $ 53.702. First of all, let us understand that while IRR was a rate, NPV is a number, or a sum. What does this sum imply? The implication of NPV will depend upon what was the discounting rate.

In Example 6, if we were to compute the IRR, the would be 13.63%. We discounted these cashflows at a discounting rate of 10%, and the NPV came to $ 53.702. If 10% represents the cost of capital or the cost of funds of the investor, then, the profit or margin inherent in the investment is 3.63%. Measured today, the value of this profit is $ 53.702.

NPV is not necessarily the profit – NPV is a measure of net value of a deal. If the discounting rate is the cost of funds, then NPV captures the monetary value of the profit, measured upfront. But there are several choices for discounting rates – we may use opportunity cost, risk free rate, etc.

Factors on which NPV would depend:

NPV is the accelerated value of the spread of the IRR in the series of cashflows over the discounting rate. To take an analogy, if I am selling something, the amount of profit depends upon what the selling price is, what the cost price is, and what is the quantity sold. NPV arises on an investment that is recovered over a period of time. Hence, there will some more additional factors that would enter here. Hence, the following are the factors on which NPV depends:

- **IRR inherent in the cashflows:** This is almost too obvious. Usually, higher the IRR in a deal, higher is the NPV at a given discounting rate.
- **Discounting rate**: This is another obvious factor. Higher the discounting rate, lower would be the discounted value. By itself, choice of a proper discounting rate for discounting the cashflows is a very significant part of NPV analysis. Some quick rules relating to the choice of discounting rate are as follows (more discussion on this follows):
    - **Objective of the analysis:** Mostly, the objective of the analyst is to find the net profit of a proposal, or the net value of several competing proposals. Hence, usually, the cost of capital, cost of borrowing, marginal cost of borrowing, or opportunity rate are used as discounting rates. The idea is to choose a benchmark rate appropriate to analyze the worth of the given proposal.
    - **Consistency of comparison:** If two or more alternative proposals are being compared, it is quite understandable that the discounting rate used to discount the cashflows of the alternative proposals must be the same. However, it is not correct to say – as long as I am using the same discounting rate for both the proposals, how does it matter what rate am I using. Higher discounting rates tend to suppress the value of cashflows that take place later in time. Hence, if the chosen rate is unduly high or unduly low, the analysis may be biased.
    - **Risk free and risk adjusted discounting rates:** If the cashflows are risk-free, that is, free of default risk (for example, treasury cashflows), a risk-free discounting rate is used. As the cashflows become more and more uncertain, the discounting rate is revised upwards – this is called *risk-adjusted discounting rate*. We will return to this point later.
    - **Pre-tax and post-tax discounting rates:** As a general rule, for discounting pre-tax cashflows, a pre-tax discounting rate (for example, pre-tax cost of capital) should be used. For discounting post-tax cashflows, a post-tax discounting rate should be used.
- **Size of investment**: As NPV is a quantity and not a rate, the size of the NPV will depend upon the ticket size, that is, the sum of money involved in the given

transaction. For example, if all the numbers in Example 9 were multiplied by 1000, the resulting NPV will also 1000 times of what our computation showed.

- **Tenure of investment**: Not only will the value depend upon how much is the size of the transaction, but also over how long tenure is the investment recovered. If a sum of $ 1000 is invested at a certain spread for 5 years, obviously it would earn more profit than if the same amount is invested for the same spread for 3 years.
- **Structure of repayment**: The structure of repayment also has the impact of elongating or shortening the investment horizon. For instance, if cashflows are scanty in the beginning, and heavier towards the end, it would mean money remains locked for a longer term. This would have the impact of increasing the net present value.

We take some examples of NPV computations to make these points obvious.

Example 7

Suppose I have $ 1000 to invest and I have two alternatives: (a) invest in a loan that pays $ 300 every year for 5 years, yearly in arrears; (b) invest in a loan that pays $ 270 every year for 5 years, yearly in advance. Suppose my cost of capital is 10%. Which of the two deals is preferable for me?

Note the question – which of the two alternatives is preferable? We have two significant tools of analysis – IRR and NPV. IRR tells us the rate of return, and if we compare that with the cost, we have the spread. The guiding rule of choosing between two competition proposals seems to be: the more the spread in the deal, the better the deal. NPV as a tool of analysis sums the spread into a number, and tells us what is the monetary value of the profit that the deal entails. Hence, one might have another potential rule for choosing between two competing deals: the more the NPV in the deal, the better the deal.

Let us apply both the tools of analysis to our example to get to some conclusion. The results of the computation are shown in Excel spreadsheet Chapter TVM Example 10 Net present values-2.

We first compute the IRRs. We have used the Rate function to compute IRRs. Option (a) gives an IRR of 15.238% and option (b) gives an IRR of 17.740%.

Next, we compute the NPVs of the two deals. We may use either the NPV function, or the PV function. We have used the NPV function here – hence, the cashflows have been written as they take place. In case of option (a), it is straight forward. In case of option (b), as the payments are annually in advance, the first payment is received immediately as the outflow takes place. Hence, the first payment is adjusted against the outflow, and there are 4 payments in future.

The NPV of option (a) comes to $137.24 and that of option (b) comes to $125.86.

The dilemma is clear – going by IRR, option (b) is better, and in fact, substantially better with returns being higher by more than 250 basis points. Going by NPV, option (a) produces a better NPV.

However, once we take a look at the cashflows of the two deals in the Excel sheet, the confusion may become clear. In option (a), I am investing $ 1000, and recovering it over 45 years. In view of the amounts received in advance, in option (b), I am investing $ 730, and recovering that over 4 years. As we have noted earlier, NPVs are affected by the size of investment as well as the investment horizon. While option (b) is surely more profitable, it gives lesser profit because there is lesser investment needed in it, and for a lesser time. Since $ 270 is received immediately, theoretically, I might invest that money also, and earn on it. Considering such reinvestment opportunities, the NPV on option (b) might also be higher. However, option (b) is surely more profitable than option (a).

In Example 7, the IRRs of the two cases were different – hence, it was easy to decide. We another example below, where the difficulty rises to a new level.

Example 8

Let us suppose I have $ 1000 to invest and I have the following 3 optional cashflow structures:

Years	Option 1	Option 2	Option 3
0	-1000	-1000	-1000
1	0	$263.80	400
2	0	$263.80	350
3	0	$263.80	200
4	0	$263.80	200
5	1610.51	$263.80	97.03

The 3 optional structures have been designed to be different. Option 1 is totally back-heavy – there are no cashflows over the term, and there is a bullet payment at the end. Option 2 pays equal instalments over the term. Option 3 is front heavy, with more cashflows in the beginning than towards the end.

Let us compute the NPVs of these 3 deals at three different discounting rates: 8%, 10% and 12%. The results of the computation are shown below:

NPVs at	Option 1	Option 2	Option 3
8%	96.09	53.27	42.24
10%	0.00	0.00	0.00
12%	-86.15	-49.07	-39.33

We first observe that at 10% discounting rate, the NPVs of each of the 3 options is zero, which means the IRR of the 3 options is 10%. This means we should be indifferent between the 3 proposals.

At 8% discounting rate, Option 1 produces maximum NPV, more than double that of Option 3. If the IRR of the 3 deals is the same, their spread at an 8% discounting rate must also be the same, and yet the NPVs are lot different.

We do a third analysis, at 12% discounting rate. Here, the discounting rate is higher than the IRR itself. If the discounting rate is the cost of money, there is a loss in either of the 3 options. But at that rate, the loss is the maximum for option 1, and the minimum for option 3. So, if loss minimization is the objective, option 3 must be selected.

So, we have pointers in three different directions – the IRR suggests that the 3 proposals are indifferent. At 8% discounting rate, we would love option 1 as it maximizes the profit. At 12% discounting rate, we will go for option 3, as it minimizes the loss.

To be able to make sense out of these 3 conflicting pointers, we need to understand what the NPV analysis is doing. As mentioned before, NPV is the absolute amount of profit or net value, and is therefore, affected by the structure of the cashflows. While we are using a discounting rate of 8%, we have a spread of 2 % (IRR being 10%) in each of the 3 deals. As it is profitable to invest, the profits will be maximized if we invested for a longer term. In option 1, the entire amount of $ 1000 remains locked for 5 years, while the other two options are amortizing, with Option 3 being amortizing faster. Obviously, therefore, the NPV is maximum where the average term for which the money remains invested is the longest.

While discounting at 12%, we must understand that if the cost of money is 12%, it is loss to do either of the 3 deals. If there is a loss, the loss is minimized by the proposal where the exit is the fastest – as is the case with option 3. Option 3 has the longest duration – hence, the loss is the maximum in case of option 3.

So ultimately, which deal do we choose? The rate of profit of each of the 3 deals is the same, as indicated by the IRR. They have different profits because one churns money out quickly (option 3), while another keeps money invested until maturity. Profits are a function of how much is invested, and for how long. But if profit is lower in an option that churns money out faster, such money can be reinvested, and the profit lost by money churned out may be restored. Hence, the 3 deals will be indifferent if the rate of return that they produce is similar to the reinvestment rate, that is, rate of return produced by other opportunities. However, if the rate of return of the 3 options is higher than most other opportunities, that is, the reinvestment rate is lower than the IRR of the 3 options, then, obviously, option 1 is the best as I am locking my money for the longest duration.

Is NPV a tool of comparison?

The substance of the above discussion is that each of the factors that we had listed above have a bearing on NPV, and therefore, one cannot choose between projects having different ticket sizes, different tenure, different payback structure, etc by comparing the NPVs. Does that mean NPV has very limited value in choosing between mutually exclusive projects? Not really, but stand-alone NPV does not say much. NPV has to be used in conjunction with something like IRR or duration to make it analytically important.

Use of discounting rate in NPV computation:

One of the most critical questions in applying the NPV method to analysis is – what discounting rate to use for discounting the cashflows. There is no uniform answer to this question. The analyst must understand the purpose of the analysis and the nature of NPV, discussed above. NPV is the present value of difference between the discounting rate and the rate of inherent in the cashflows (assuming the cashflows have a rate of return inherent). The discounting rate is like the measuring yardstick – the analyst must ask – what do I compare these values with?

A few bullets below seek to explain the appropriate rate to use for discounting of cashflows:

- Let us say I am analyzing the cashflow of Project A, in which I would invest. I have the opportunity of investing in Project B that would give me a rate of return of 10%. In this case, 10% discounting rate used for Project A would serve the purpose of the analyst. The rate of return in Project B forms the opportunity rate or opportunity cost for Project A.
- In the same example as above, if we were analyzing the cashflows of both Project A and Project B, and the funding of the 2 projects is to come from a certain combination of debt and equity, then the weighted cost of capital of the sources of funding will form an appropriate discounting rate. The NPV at that rate indicates whether the two projects are leading to a profit on cost of financing or not.
- Let us suppose Project A and B in the example above will be funded from the internal resources of the company. In such case, weighted average cost of capital for the company will be an appropriate discounting rate. Note that there is a difference cost of financing of a particular project, and the cost of capital of the company. The cost of capital of the company is the weighted average of all long term sources of funding the balance sheet of the entity. The cost of financing a particular investment refers to the cost of sources used for financing that particular project.
- If the cashflows in question are the residual cashflow from a project, net of interest and financing costs, then the appropriate discounting rate is the return on equity or return on economic capital.
- For example, if I am considering whether investing in a bond will be better than investing in a risk free security, or which of the two or more fixed income options are better, I can use the risk free discounting rate.
- A very important point to note is that if the cashflows being discounted are post-tax cashflows, then a post-tax discounting rate (for example, post-tax cost of capital) should be used. If the cashflows are pre-tax cashflows, then the discounting rate should also be a pre-tax discounting rate. For example, the post-tax cost of capital may be converted into its pre-tax equivalent [by dividing by (1-tax rate)] and then used as the discounting rate.

- The discounting rate is often also adjusted to take care of the riskiness of the cashflows. This is called *risk adjusted discounting rate*. See below. However, if the riskiness of the cashflows has already been captured by computing the expected value of the cashflows, then the discounting rate to be used is the discounting rate without the impact of risk-adjusting, that is, risk-free discounting rate.
- Another very significant point is that the discounting rate does not have to be *static* – it may be a variable discounting rate. For example, if the rate of return on risk free securities (government treasuries, the rate is also called the *yield curve*) is used as the discounting rate, it is well known that the rate is not the same for different tenures. Hence, cashflows occurring at different points of time may be discounted at different discounting rates.

Incorporating riskiness of cashflows:

F uture cashflows may quite often be uncertain. The uncertainty of future cashflows in dealt with in one of the two ways in time value of money analysis:

- **Risk adjusted discounting rate**: the discounting rate used for discounting is adjusted upwards to incorporate the riskiness of the cashflows. This is a standard method used for discounting streams of cashflows, as the other method, discussed below, requires estimation of probabilities of cashflows, which is mostly arbitrary. It is not possible to exactly find the amount by which the discounting rate should be adjusted to reflect the risk. Hence, the following are the options for risk-adjusting the discounting rate:
    - **Use of beta of the cashflows**: In capital asset pricing model (CAPM) jargon, if the beta of the industry to which the project or the cashflows belong, is known, then, the rate of return used for discounting the cashflows may be adjusted upwards based on the *beta* of the cashflows. Note that the beta relevant here will be the asset beta, and not equity beta. The equity beta may be used as an input, leverage being the other required input, to estimate the asset beta.
    - **Use of return on equity, or weighted average cost of capital:** As discussed elsewhere, if the cashflows are forming integral part of the overall cashflows of an entity, the discounting rate is the weighted average cost of capital of the entity, or the weighted average cost of financing a project, as may be appropriate. However, if the cashflows are residual profits of a project, net of external financing costs, then the cashflows reflect the risk of leverage too, and therefore, the return of equity, which is substantially higher than cost of capital, is used as the discounting rate.
- **Use of expected values**: Assuming it is possible to estimate or put values to the probabilities of different cashflows in different scenarios, then the expected value of the cashflows may be computed. The expected value **E** may be expressed as:

$$E = \sum CF_i \ p_i : \sum p_i = 1$$

where

CF_i are cashflows in different probable scenarios

p_i is the probability of the scenario.

Example 9

Let us suppose I have $ 1000 to invest and I have the following 3 three alternative scenarios of cashflows from the investment:

Years	Scenario 1	Scenario 2	Scenario 3
Probability	25%	50%	25%
0	-1000	-1000	-1000
1	200	$263.80	180
2	250	$263.80	190
3	250	$263.80	200
4	275	$263.80	220
5	280	$263.80	270

To compute the present value of the investment, the analyst first ensures that the discounting rate being used has not been risk-adjusted for the riskiness of the cashflows. It does not have to be risk-free discounting rate, meaning rate of return on government treasuries, but it should no be adjusted upwards for the risk of the cashflows. For example, if the cost of capital is being used as the discounting rate, the same should not be risk-adjusted for the risk of the project. Rather, if one or more inputs of cost of capital (for example, cost of equity) have been affected by the risk of the business (for example, the beta of the industry), then the effect of the same should be removed.

Let us say, we decide to use a discounting rate of 8%. We may compute the Expected value of the NPVs (that is, compute NPV of the cashflows in the 3 scenarios, and then sum up the product by multiplying with the probabilities), or the NPV of the expected values (that is, compute expected value of cashflows in each year, and then discount the expected values) – the result will be the same. The result is shown below:

Years	Scenario 1	Scenario 2	Scenario 3	Expected values
Probability	25%	50%	25%	
0	-1000	-1000	-1000	
1	200	$263.80	180	226.9
2	250	$263.80	190	241.9
3	250	$263.80	200	244.4
4	275	$263.80	220	255.65
5	280	$263.80	270	269.4
NPVs	$990.67	$1,053.28	$833.79	$982.75
Expected value of NPVs			982.7549	

Decision trees:

Yet another way of incorporating and evaluating different scenarios is to use decision trees. Here, there are two or more scenarios at the inception, and each scenario in turn leads to two or more scenarios, leading to a kind of a tree with branching, splitting into sub-branches, and so on. Consider the following:

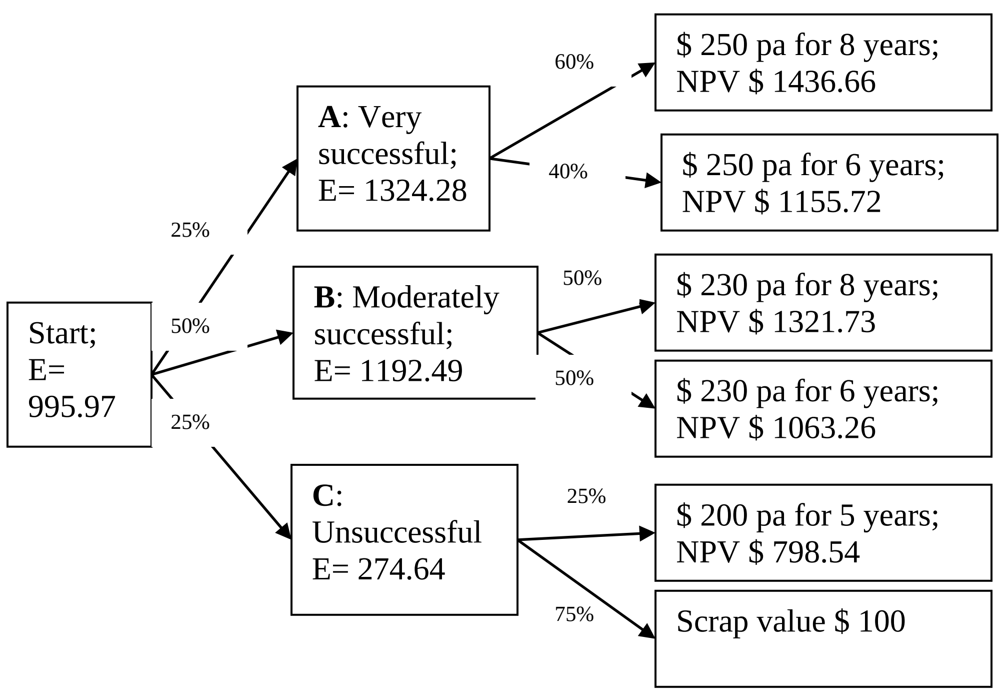
Example 10

Let us suppose I have $ 1000 to invest in a Project, and I have the following 3 three alternative scenarios (probabilities in brackets) : the Project is very successful (25%), the Project is moderately successful (50%), and the Project unsuccessful (25%). The 3 scenarios are further analyzed as follows:

- If it is very successful, there is a chance (60%) that it gives cashflows of $ 250 per annum for 8 years, or a chance (40%) that it gives a cashflow of $ 250 for 6 years.
- If it is moderately successful, there is a chance (50%) that it gives cashflows of $ 230 per annum for 8 years, or a chance (50%) that it gives a cashflow of $ 230 for 6 years.
- If it is unsuccessful, there is a chance (25%) that it gives cashflows of $ 200 per annum for 5years, or a chance (75%) that it is scrapped right away with a scrap value of $ 100

Say we use a discounting rate of 8 % (once again, this rate is risk-free discounting rate, as the risk is being captured in the scenarios).

The scenarios above are captured in the following decision tree:



To evaluate the decision tree, we first evaluate the branches on the right-hand side and compute the value of each branch. Then, we come to the *nodes* where these ultimate branches began, and compute the value at each node. For example, the value at Node A is the expected value of the two branches branching out at this node, by applying the respective probabilities to their likely values. Same way, we compute the expected values at each of the nodes, and then, compute the value at the starting point.

Measures of risk: measures of dispersion:

The expected value computed above is the weighted average value, weighted by the respective probabilities of the scenarios. However, behind every average is the dispersion from the average. The idea of dispersion is the real picture of risk. If you are trying to walk through a river, what matters it not so much the average depth of the river, but the deviations from the average. Likewise, the expected value fails to give an idea of how far flung the values are from the average. For example, two projects might both have expected value of $ 1000, but the underlying volatility, that is, the dispersion of values away from the mean, may greatly differ. Hence, the riskiness of the two projects may be widely different. Therefore, expected value does not give sufficient information about the risk.

The central idea in trying to understand the risk of cashflows is to understand the volatility of the values. There are several measures used to measure the risk, illustrated below.

Example 11

Let us suppose I have $ 1000 to invest in a Project. I have done the NPV analysis, and I have the following 3 three alternative scenarios, with their respective probabilities:

Scenario	NPVs	Probability
1	-200	30%
2	150	50%
3	250	20%

Notation:

X_i : the *i*th value

$\overline{X} = \sum X_i P_i$

where P_i is the probability of the *i* th value.

- **Expected value**, or mean value is 65.
- **Range** captures the difference between the highest and possible values in the scenarios. The range is an absolute reflection of risk, and not a relative measure. In the above case, the range is 450 (250 – (-200)).
- **Mean absolute deviation (MAD)** is the mean of absolute value of the differences between the respective values and the mean, multiplied by the probabilities.

  MAD = $\sum \left|(\overline{X} - X_i)\right| P_i$

In the present case, the MAD is computed below. Note that the number 159 is obtained by summing up the product of the values in col 4 with the probabilities in Col. 3

Scenario	NPVs	Probability	Abs Deviations
1	**2**	**3**	**4**
1	-200	30%	265
2	150	50%	85
3	250	20%	185
Expected value		65	159

- **Standard deviation (σ)** is one of the most commonly used methods of dispersion in statistical analysis. This is computed by squaring the differences of the values from the mean, multiplying the same by respective probabilities, summing up the result, and then finding the root of the sum.

$$\sigma = [\sum (X_i - \overline{X})^2 P_i]^{1/2}$$

In the example above, standard deviation has been computed as follows:

Scenario	NPVs	Probability	Abs Deviations	Deviations	squared	product of prob
1	**2**	**3**	**4**	**5**	**6**	**7**
1	-200	30%	265	-265	70225	21067.5
2	150	50%	85	85	7225	3612.5
3	250	20%	185	185	34225	6845
Expected value		65	159			31525
standard deviation						177.5528

- **Variance (σ^2)** is simply the square of standard deviation. In the present case, the variance is 31525.
- **Coefficient of variance** is the standard deviation, expressed as a coefficient of the mean, that is, standard deviation divided by the mean. In the present case, the same is 2.73.
- **Semi-variance** is the same as variance, with the difference that here we look at only the risk of downward variation, that is, ignoring cases where values are above the mean. In the above example, there is only one case where the value is below the mean – the deviation in that case being -265. So, we will square the same, and multiply it by the probability. This is the semi-variance, in the present case 21067.5

Assessing risk by simulation:

Where a project or investment has different outcomes or scenarios, a simulation run tries to simulate, that is, mimic the reality about the outcomes of the project. The reality is uncertain, and uncertainty is best captured by random numbers. The use of random numbers to simulate the likely values of a project is given by the following algorithm. Let us take a simple project which has the following likely values:

Outcome	Probability	Cumulative Prob
1	**2**	**3**
3000	10%	10%
5000	25%	35%
7000	45%	80%
9000	20%	100%

- The cumulative probabilities, given in in Col 3, are values between 0 and 1. [It would not be difficult to understand why we do not use the marginal probabilities given in Col 2 – as they do not ascend from zero and add up to number 1. As we are slotting the probabilities against random numbers, the objective is served by comparing the random numbers with cumulative probabilities].
- Random numbers are also randomly generated numbers between 0 and 1.
- By definition, random numbers are uniformly distributed. That is to say, the chances of getting a number between 0 and 0.1 is roughly 10%. The chances of getting a number between 0 and 0.35 is roughly 35%, and so on.
- The probabilities in Col 3 represent the thickness of the chances of getting the values given in Col 1. For example, there is a 25% chance (see Col 2) that the outcome is 5000. If we were to look at random numbers, there is, likewise, a 25% chance that the number is between 0.1 and 0.35. In other words, if we select random numbers, and we find that the number is between 01. and 0.35, we may say this corresponds to the probability of getting an outcome of 5000.
- Likewise, each random number in the simulation runs is slotted against the probabilities in Col 3., and we take the corresponding value in Col 1 as the likely value.
- In order for any simulation run to be reliable, there must be lots of simulations done. Manually, this is hard to achieve. However, on standard simulation engines, or on Excel, it may be possible to assimilate the results of hundreds of thousands of simulations.
- The mean value of the simulation runs represents the most likely value of the project.
- Expectedly, if sufficient number of simulation runs are taken, the likely value of the project will be the same as expected value produced by multiplying the likely outcomes by the probabilities. This goes by the very nature of random numbers. However, the advantage with random numbers is that we can randomize several relevant parameters at the same time – for example, not just the final outcome of the project, but several input variables or other relevant parameters may be randomized. This would be difficult to achieve in simple expected value computations.

Capital rationing:

Capital rationing refers to the allocation of limited capital resources to different competing projects, under the assumption that the firm cannot invest in all of them. So, if we assume that there is a capital constraint, and if the firm may invest in several competing projects, not all of which may be *mutually exclusive*, then, the objective is to choose such combination as maximizes the net present value to the firm.

Intuitively, the exercise is simple – the criteria for choosing projects is the *profitability index*, that is, the NPV of the project divided by its capital outlay, or the NPV per dollar of capital outlay. Needless to say, the project with the highest profitability index is the one that must be chosen first. [One important rider here is that we cannot ignore the duration of the project. To be more precise, one must say, profitability index, divided by duration, should be the decision criteria, because a project that achieves higher NPV by keeping the investment locked for a longer duration, is not necessarily preferable].

Since capital investment projects are not divisible (that is, one cannot undertake 50% of a project), one would have to list out all possible combinations that fall within the capital constraint, and choose the one that maximizes the NPV.

Inflation-adjusted cashflows:

One of the issues in capital budgeting is to incorporate the impact of inflation on future cashflows. Basically, the discounting rate or rate of return includes an element of inflation too. It is common knowledge that interest rates are partly to compensate for inflation, and partly to provide a *real rate of interest.* The components of cost of capital, which is often used as the discounting rate, also capture the impact of inflation, as lenders seek compensation for inflation too, and so also do equity shareholders. Hence, prima facie, the impact of inflation is already taken care of in the discounted vales.

There is yet another reason cited as to why inflation is not captured as a specific factor in capital budgeting – inflation has common impact on both the required rate of return, and on the projected cashflows, and hence, tends to cancel out itself. For example, if future revenues were expected to grow 10% because of inflation, and the discounting rate was also to be adjusted upwards to compensate for inflation of 10%, then the impact of inflation gets neutralized as both the numerator and the denominator in the estimated cashflows are affected by a common rate of inflation.

However, if future cashflows have been projected under assumption of price fixity, then incorporating the impact of inflation leads to some unique calculations. There are cashflows affected by inflation, and there are cashflows not affected by inflation. For example, in case of capital budgeting decisions, depreciation will be based on historical cost of the asset, and will not be affected by inflation. There may be other elements of costs – say, rent, which is fixed and not affected by inflation.

Hence, inflation-adjusted cashflows, and inflation-adjusted discounting rate, may not exactly neutralize. Therefore, it may be important to incorporate the impact of inflation. The steps in doing the same are as follows:

- Expected cashflows which are inflation-indexed – say, selling prices, should be revised upwards by incorporating the inflation rate. Incorporating the impact of inflation is the same as compounding of the cashflows. The rate of inflation is often expressed as annually compounded rate.
- Needless to say, those cashflows that are not affected by inflation, such as fixed expenses or revenues, capital allowances, etc. are taken without incorporating the impact of inflation.
- Having thus found the inflation-adjusted cashflows, there are two approaches:
    - The discounting rate may not be inflation-adjusted rate. In this case, the projected cashflows should be deflated by removing the impact of inflation. This is done in the same way as present valuing the cashflows – that is, by dividing the inflation-adjusted cashflows (this includes those that are not indexed with inflation) by (1+inflation rate)^n. Such inflation-deflated cashflows are discounted by usual discounting rates to obtain the present value.
    - Alternatively, if the discounting rate already captures the impact of inflation, then the inflation-adjusted cashflows are discounted at that rate.